Minimum Weight Potentials for Stiffened Plates and Shells

R. F. Crawford*
Martin Company, Baltimore, Md.
AND
A. B. Burns†
Lockheed Missiles and Space Company, Sunnyvale, Calif.

Existing principles and concepts of minimum weight analysis are used to derive detailed design information for stiffened, axially compressed cylinders. Results from similar studies also are presented for hydrostatically compressed cylinders, wide columns, compression panels, and multiweb box beams in bending. A common approach can be used in all analyses which generally results in an efficiency equation of consistent and convenient form. Minimum weights are compared in all cases, showing the relative efficiencies of various stiffening arrangements. The effect of material properties on structural efficiency also is demonstrated.

Introduction

The principles of minimum weight analysis and design have been established for some time. However, the need exists for further exposing and developing the generality of these principles and showing their application to a greater variety of structural members. For the present paper, these principles were used to derive detailed minimum weight design information for plates and shells of various detailed stiffening arrangements and loadings. The loading-component combinations investigated are axially and hydrostatically compressed cylinders, wide columns, flat compression panels, and multiweb box beams in bending.

Comparisons between optimum designs of various detailed configurations of each loading-component combination that was considered generally indicated that conventionally stiffened structures are more efficient than unstiffened structures, and that sandwich configurations are the most efficient. These observations could be made without considering specific material properties, which, by themselves, strongly influence structural efficiency. Of the metals investigated, beryllium proved the most efficient for all lightly loaded applications—from room to moderately elevated temperatures. For lightly loaded (low stress) applications, only density and modulus affect efficiency, whereas for high plastic stress applications, efficiency is virtually independent of the modulus. Here, for the latter case, materials having a high strength-to-density ratio are shown to be the most efficient.

Review and Discussion

The concepts of loading indices and efficiency factors, which have proved very useful in structural efficiency analysis, are relatively recent. The development of these concepts is attributed to Zahorski.[1] They have been used very effectively by Farrar,[2] Shanley,[3] Gerard,[4] Catchpole,[5] and others.

The loading index concept is applied in a minimum weight or efficiency analysis by expressing that quantity to be minimized (weight) or maximized (stress) in terms of the prescribed dimensions and load. Those prescribed quantities are combined in such a manner that the resulting loading index has dimensions of force divided by length squared. In the case of a column, the loading index is P/L^2, where P is the load to be transmitted and L is the length over which it is to be transmitted. Zahorski, Farrar, and others have developed efficiency equations for stresses in wide columns in which they identify as efficiency factors those quantities that multiply the product of the modulus and loading index, i.e.,

$$\sigma = F[\eta E(N_x/L)]^{1/2} \tag{1}$$

where F is the efficiency factor, η a plasticity reduction factor, E Young's modulus, N_x the loading per unit width, and L the unsupported length. The efficiency factor is a function of certain geometric proportions of the construction and can be varied independently to maximize F. Stress is maximized accordingly.

In the present work, the efficiency equation is used in a more general form, which applies to various types of structural components. First, the loading index is nondimensionalized by dividing it by the effective modulus ηE. The resulting loading-material index isolates both material effects—including plasticity—and loading index in a single index. Secondly, since one often is concerned more with minimum weight than with maximum stress, the efficiency equation is developed in terms of a weight index instead of a stress. The weight index is useful especially in the case of box beams, where maximizing cover stress generally would not result in minimum weight. All efficiency equations in the present work are therefore in the general form

$$\text{loading-material index} = \text{efficiency factor (weight index)}^n \tag{2}$$

The efficiency factor is designated $\mathcal{E}$ and remains a function of certain geometric proportions of the construction under consideration. These may be treated as independent variables in maximizing $\mathcal{E}$. The weight index usually is designated as the nondimensional quotient of a quantity $\bar{t}$ divided by the same specified dimension of the structure as is used in forming the loading-material index. The quantity $\bar{t}$ is an effective thickness for purposes of weight calculation. The quantity n is an exponent, the value of which depends on the particular component. Minimum weight or maximum structural efficiency is obtained when $\mathcal{E}$ is maximized. The values of geometric proportions affecting $\mathcal{E}$, whether they are optimized or not, determine the efficiency of a given construction.

The basic principle used in arriving at Eq. (2) for a given structural element of this category is that, for optimum design, at least the two lowest modes of instability are critical under the applied loading. One facet of this principle is illustrated by the tubular column problem. For a given amount

Presented at the ARS Launch Vehicles: Structures and Materials Conference, Phoenix, Ariz., April 3–5, 1962; revision received December 17, 1962. This research was supported by the U. S. Air Force under Contract No. AF33(616)-6905 and Lockheed Independent Research funds.

* Senior Staff Engineer, Space Systems Division; formerly Research Specialist, Lockheed Missiles and Space Company. Member AIAA.

† Research Specialist.

of material and length, the Euler load is increased as the column's diameter is increased, until the walls become unstable as a thin shell. That diameter is optimum, so that any further increase in diameter only would lower the column's strength.

Another facet of the principle is illustrated in the optimum design of a multiweb beam in pure bending. If the webs are designed to be critical, both for carrying flexure-induced crushing forces and for providing stiffness necessary to stabilize the compression cover, then the box beam generally will be less efficient than one designed only for the more demanding (weight-wise) of the two web design criteria.

In stiffened structural components, such as plates and cylinders, the two modes equated are usually local and general instability of the component. Local instability is characterized by local displacements normal to the planes of the stiffener and skin elements during buckling, while lines joining the elements remain undisplaced. General instability is characterized by general displacement of the composite during buckling.

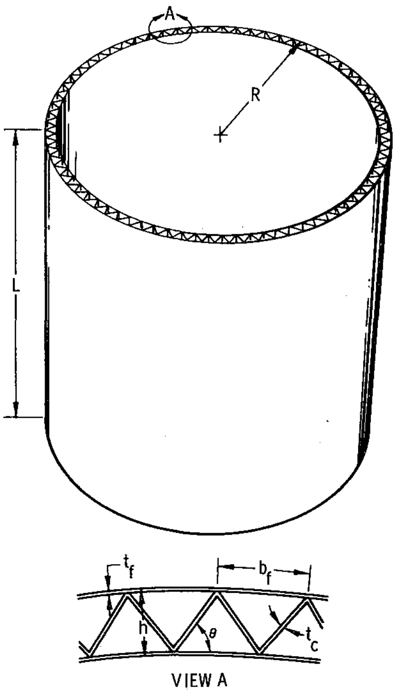


Fig. 1 Geometry of the truss-core sandwich cylinder

In some cases, efficiency analyses do not yield the simple form of Eq. (2), but optimum proportions still can be determined from the more complex formulation. In these cases, optimum proportions are dependent on the loading-material index, and charts showing the relationships are required. However, even in these cases, the resulting efficiency charts of weight index vs loading-material index can be approximated closely by equations of the form of Eq. (2). Certain geometric proportions are virtually constant for maximum efficiency, regardless of loading-material index in these cases.

Following is an efficiency analysis of long, axially compressed, truss-core sandwich cylinders, where the methods of this type of analysis are exemplified. Subsequently, the results of efficiency analyses of the remainder of the loading-component combinations mentioned in the Introduction are given and compared to show relative efficiencies among several detailed configurations of each loading-component combination. Because of space limitations, only final results of efficiency analyses for the various loading-component combinations are given, but the source of each analysis is referenced. Finally, effects of material properties on structural efficiency are presented.

Analysis of Long, Axially Compressed, Truss-Core-Sandwich Cylinders

This analysis is that of Crawford and Stuhlman[6] and pertains to sandwich cylindrical shells whose corrugated core elements extend in the axial direction, as shown in Fig. 1. The basis of the analysis is small deflection theory, which appears reasonable in this case, even though it is known to be inaccurate for monocoque cylinder design—except at very low ratios of radius to wall thickness.

In the small-deflection analysis for sandwich cylindrical shells of Ref. 7, the stability equation is quite similar to that for monocoque shells, since the critical stress is directly proportional to the ratio of sandwich thickness to shell radius when effects of core shear stiffness are neglected. Sandwich cylinders, which have larger wall thicknesses than monocoque cylinders of equal load-carrying ability and radius, fall into a thickness-to-radius ratio range that tends to make general instability predictable by small deflection theory over a significant range of practical applications. Test data that confirm this tendency have been obtained and are presented later in this section.

In the following analysis, the sandwich facings are taken to be of equal thickness. Effects of curvature are included in the general instability analysis but are neglected in the local instability analysis. The application of the information is limited to long cylinders, defined as having $(R/h)(L/R)^2 > 5$.

General instability of a long truss-core sandwich cylindrical shells subjected to uniform axial compression, as predicted by Stein and Mayers,[7] can be approximated closely by

$$\frac{\sigma_{crG}}{\eta_G} = \frac{2}{\bar{t}}\left(\frac{2t_f E D_x}{R^2}\right)^{1/2} \tag{3}$$

The quantity η_G is a plasticity reduction factor for general instability.

The quantity $\bar{t}$ may be expressed in the following form:

$$\frac{\bar{t}}{R} = 2\frac{t_f}{b_f}\frac{b_f}{R}\left(1 + \frac{1}{2}\frac{t_c}{t_f}\frac{1}{\cos\theta}\right) \tag{4}$$

The flexural stiffness D_x is given in Ref. 8 in the form

$$\frac{D_x}{D_f} = \frac{3}{2}\left(\frac{b_f}{t_f}\right)^2 \tan^2\theta\left(1 + \frac{1-\nu^2}{6}\frac{t_c}{t_f}\frac{1}{\cos\theta}\right) \tag{5}$$

where

$$D_f = [Et_f^3/12(1-\nu^2)] \tag{6}$$

and ν = Poisson's ratio.

Combining these equations results in the following expression for general instability:

$$\sigma_{crG} = \frac{6\eta_G D_f (b_f/R)\tan\theta}{t_f^3[1 + \frac{1}{2}(t_c/t_f)(1/\cos\theta)]} \times \left[(1-\nu^2)\left(1 + \frac{1-\nu^2}{6}\frac{t_c}{t_f}\frac{1}{\cos\theta}\right)\right]^{1/2} \tag{7}$$

Local instability of truss-core sandwich elements, including the effects of coupling between adjacent elements, is predicted in Ref. 9 as

$$\sigma_{crL} = k_x\pi^2\eta_L D_f/b_f^2 t_f \tag{8}$$

where k_x, the buckling coefficient, is dependent upon the ratio t_c/t_f and θ, and η_L is the plasticity reduction factor for local instability.

As stated, optimum proportions of the sandwich result when it is designed to have equal critical stress in the two modes of instability. Accordingly, Eqs. (7) and (8) are equated, which results in the following expression for the

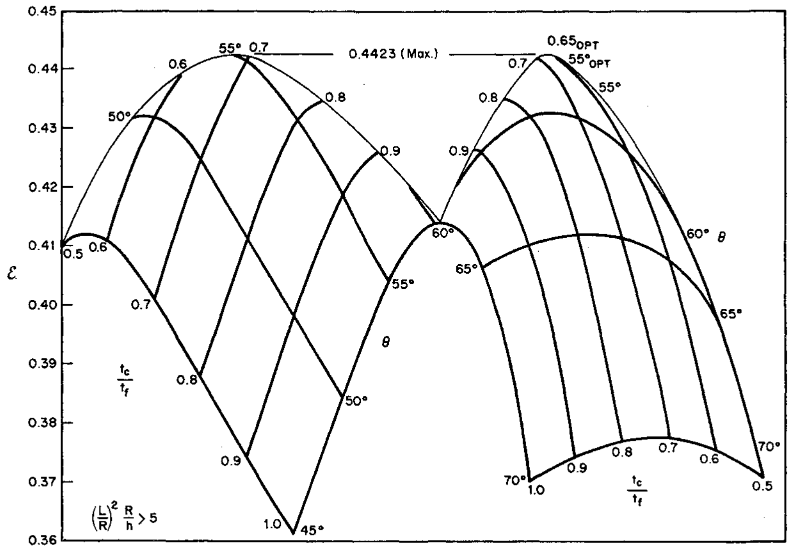


Fig. 2 Efficiency factors for truss-core sandwich cylinders under uniform axial compression

ratio b_f/R:

$$\frac{b_f}{R} = \frac{\eta_L}{\eta_G}\left(\frac{t_f}{b_f}\right)^2 \frac{k_x\pi^2}{6\tan\theta} \times \frac{[1+\frac{1}{2}(t_c/t_f)(1/\cos\theta)]}{\{(1-\nu^2)\{1+[(1-\nu^2)/6](t_c/t_f)(1/\cos\theta)\}\}^{1/2}} \quad (9)$$

Eq. (8) may be expressed in the form

$$\frac{N_x}{R\eta_L E} = \frac{k_x\pi^2}{12(1-\nu^2)}\left(\frac{t_f}{b_f}\right)^2 \frac{\bar{t}}{R} \quad (10)$$

where N_x is the compressive loading per unit of circumferential length.

One equation involving the loading-material index, $N_x/R\bar{\eta}E$, the weight index $\bar{t}/R$, and the condition of Eq. (9) is required. For this purpose, Eqs. (4, 9, and 10) are combined, and the geometric ratios t_f/b_f and b_f/R are eliminated:

$$\frac{N_x}{R\bar{\eta}E} = \left[\frac{k_x\pi^2\tan^2\theta}{192(1-\nu^2)^2} \times \frac{\{1+[(1-\nu^2)/6](t_c/t_f)(1/\cos\theta)\}}{[1+\frac{1}{2}(t_c/t_f)(1/\cos\theta)]^4}\right]^{1/3}\left(\frac{\bar{t}}{R}\right)^{5/3} \quad (11)$$

where

$$\bar{\eta} = \eta_L^{1/3}\eta_G^{2/3} \quad (12)$$

The plasticity reduction factor η_G is due to von Kármán and is given by Timoshenko.[10] For the present case, von Kármán's reduced modulus is approximated by neglecting the core material. Thus

$$\eta_G = [2\eta_T/(1+\eta_T)]^{1/2} \quad (13)$$

where η_T is the ratio of the tangent modulus to Young's modulus. The local buckling plasticity reduction factor η_L is taken as

$$\eta_L = \eta_T^{1/2} \quad (14)$$

Therefore,

$$\bar{\eta} = [2\eta_T^{3/2}/(1+\eta_T)]^{1/3} \quad (15)$$

Note that Eq. (11) is in the form of Eq. (3) and may be expressed as

$$N_x/R\bar{\eta}E = \mathcal{E}(\bar{t}/R)^{5/3} \quad (16)$$

where

$$\mathcal{E} = \left[\frac{k_x\pi^2\tan^2\theta}{192(1-\nu^2)^2}\frac{\{1+[(1-\nu^2)/6](t_c/t_f)(1/\cos\theta)\}}{[1+\frac{1}{2}(t_c/t_f)(1/\cos\theta)]^4}\right]^{1/3} \quad (17)$$

As noted, minimum weight results when $\mathcal{E}$ in Eq. (16) is a maximum. Maximum $\mathcal{E}$ may be obtained by independently varying the parameters t_c/t_f and θ in Eq. (17). Values of k_x corresponding to values of t_c/t_f and θ may be taken from Ref. 9. The resulting efficiency factors are shown in Fig. 2, in which $\nu = 0.3$ was assumed. The maximum value of $\mathcal{E}$, which is seen to be 0.4423, occurs when $\theta = 55°$ and $t_c/t_f = 0.65$. The proportions are optimum, regardless of values of the loading-material index $N_x/R\bar{\eta}E$ or the weight parameter $\bar{t}/R$. Therefore, the minimum weight equation for long, axially compressed, truss-core sandwich cylinders is

$$N_x/R\bar{\eta}E = 0.4423(\bar{t}/R)^{5/3} \quad (18)$$

Additional equations necessary for design purposes are

$$t_f = \frac{\bar{t}}{2+(t_c/t_f)(1/\cos\theta)} \quad (19)$$

$$b_c = b_f/2\cos\theta \quad (20)$$

$$b_f = 0.95t_f\left[\frac{k_x(\bar{t}/R)}{N_x/R\eta_L E}\right]^{1/2} \quad (21)$$

Effects of transverse shearing stiffness have been neglected in the preceding analysis, because they are negligibly small for the geometries in the vicinity of optimum design. This conclusion may be verified by using the shear stiffnesses for these proportions, which may be calculated from the formulas in Ref. 9, in the analysis of axially loaded sandwich cylinders presented in Ref. 7.

Test data are given in Fig. 3 to show that small deflection theory for sandwich shells does appear adequate, when these sandwiches have overall thickness-to-radius ratios in the $\frac{1}{50}$ to $\frac{1}{200}$ range. Sandwiches in that range represent replacements for monocoque shells of equal load-carrying ability which would have skin thickness-to-radius ratios in the range of $\frac{1}{200}$ to $\frac{1}{1000}$. The data shown in Fig. 3 give little information on the validity of small deflection theory for the smaller ranges of sandwich thickness-to-radius ratios and give no direct information on the effect of either transverse shear stiffness or wrinkling—both of which require attention.

The ideal truss-core configuration determined has sufficient core stiffness to preclude both premature general instability

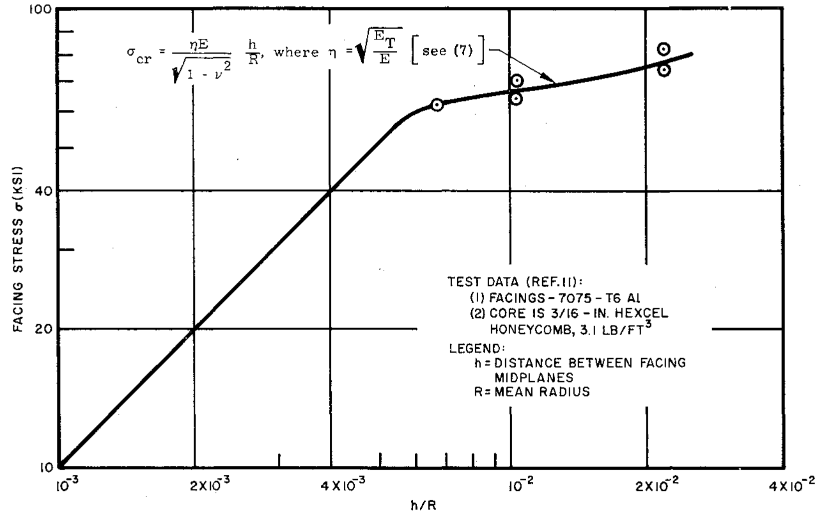


Fig. 3 Theoretical and experimental general instability stresses for circular cylindrical sandwich shells in uniform axial compression

and wrinkling, as was true of the specimens from which the data in Fig. 3 were obtained. However, practical limitations on fabrication techniques, which may introduce large bend radii or weak facing-to-core bonds, can reduce the effective core stiffnesses to values significantly below those of the ideal configuration analyzed and therefore can cause premature failure. Precautions are recommended to insure that the sandwich develops sufficient core stiffness.

Comparative Configuration Efficiencies for Loading-Component Combinations

Efficiency analyses similar to the preceding ones have been performed by various investigators for hydrostatically compressed cylinders, wide columns, compression panels, and multiweb beams. Minimum weight equations resulting from these analyses appear in Table 1. References also are listed in the table where detailed design information can be found for the respective configurations of the components tabulated. Only the references are cited for that information, since present space limitations prohibit more exhaustive treatment. In that regard, it is noted that the references cited in Table 1 are not necessarily those in which the original work was performed. This is particularly true of Ref. 12. However, that reference is cited because it summarizes results from many sources.

The various configurations for each loading-component combination are compared graphically in Figs. 4–8, using the equations of Table 1. These figures show that, for all loading-component combinations, the various configurations tend to converge as the loading-material indices increase. Thus, the weight advantages of the stiffened configurations are greatest for low values of the loading-material indices and smallest for high values of the indices.

It should be noted, particularly, that weight could be reduced potentially by more than an order of magnitude at low indices by using stiffened, rather than unstiffened, construction. Among the stiffened configurations investigated, the truss-core sandwich is the most efficient for all loading-component combinations, except wide columns. However, its superiority is seen to be more impressive in some cases than in others.

Information such as that summarized in Figs. 4–8 is quite valuable to the advance designer, because, with only the specified loading intensity and overall dimensions, he can determine readily the most suitable type of construction for his purpose, as well as its weight. He is assured in that determination that the information used represents the maximum potential efficiency for each type of construction. This type of information needs to be developed for a much wider selection of construction and loading combinations. It can be done by applying the same method and principles.

Effects of Material Properties on Structural Efficiency

So far, the effects of material selection have been divorced from this discussion, so as to emphasize effects of geometric variations. From the preceding, it can be seen that the quantity $\bar{\eta}E$ in the loading-material index for the axially compressed sandwich cylinders, or in any of the other cases summarized, embodies the effects of material properties. For long, axially compressed, truss-core sandwich cylinders, it may be evaluated for a given material according to Eq. (15).

A chart then may be made of the resulting relationship between the loading index N_x/R and the weight index $\bar{t}/R$ for the material. This type of chart has been prepared for cross-rolled beryllium sheet at room temperature (see Fig. 9). A value of Young's modulus equal to 44×10^6 psi and a value of the compressive yield stress equal to 65 ksi have been used in the preparation of this chart.

The maximum buckling stress can be assumed equal to the compressive yield stress. When this condition is plotted in Fig. 9, the result is a straight line having a positive 45° slope, to which minimum weight curves for all constructions eventually become tangent. Thus, the differences in efficiency between configurations (Fig. 4) have a practical limitation represented by the maximum attainable buckling stress for a given material. Above some value of the loading index, all configurations will be stressed at the material's maximum buckling stress and therefore will have equal efficiency. Hence, configuration selection should be based on other factors in those instances.

The straight line, left-hand portions of the curves in Fig. 9 represent elastic stresses and are a function of the Young's modulus of the material. The point where each curve deviates upward from a straight line represents a stress level in the configuration equal to the proportional-limit stress. Additional load increases the plastic stress in the configuration, until the maximum buckling stress is reached, as discussed. The more efficient the configuration, the lower the value of the loading index that marks the onset of plastic deformation. Thus, for this particular case, the most efficient configuration has the highest stress for a given applied load.

Table 1 Minimum weight equations

Loading-component combination	Configuration	Minimum weight equation	Conditions	References
Axially compressed cylinders	Monocoque	$\frac{N_x}{R\bar{\eta}E} = 7.26\left(\frac{\bar{t}}{R}\right)^{2.54}$	$\frac{L}{R} > \frac{3}{4},\ \frac{R}{t} > 100,\ t > 0.005$	(empirical)
	Truss-core sandwich	$\frac{N_x}{R\bar{\eta}E} = 0.4423\left(\frac{\bar{t}}{R}\right)^{5/3}$	$\left(\frac{L}{R}\right)^2 \frac{R}{h} > 5$	6
Hydrostatically compressed cylinders	Monocoque	$\frac{p_{cr}}{E} = 0.971\left(\frac{\bar{t}}{R}\right)^{5/2}$	$Z > 100,\ \frac{L}{R} = 1$	13
	Truss-core sandwich	$\frac{p_{cr}}{E} = 0.2961\left(\frac{\bar{t}}{R}\right)^{1.74}$	$\frac{L}{R} = 1,\ \frac{R}{h} > 100,\ \sqrt{\frac{h}{R}} < 6.75$	6
	Ring stiffened	$\frac{p_{cr}}{E} = 0.362\left(\frac{\bar{t}}{R}\right)^{1.826}$	$Z > 100,\ \frac{L}{R} = 1,\ \frac{p_{cr}}{E} > 10^{-6}$	13
Wide columns	Unstiffened	$\frac{N_x}{\ell\bar{\eta}E} = 0.823\left(\frac{\bar{t}}{\ell}\right)^3$	Loaded edges simply supported	12
	Unflanged integrally stiffened	$\frac{N_x}{\ell\bar{\eta}E} = 0.656\left(\frac{\bar{t}}{\ell}\right)^2$	↓	↓
	Zee stiffened	$\frac{N_x}{\ell\bar{\eta}E} = 0.911\left(\frac{\bar{t}}{\ell}\right)^2$	↓	↓
	Truss-core sandwich	$\frac{N_x}{\ell\bar{\eta}E} = 0.605\left(\frac{\bar{t}}{\ell}\right)^2$	↓	↓
Compression panels	Unstiffened	$\frac{N_x}{b\bar{\eta}E} = 3.62\left(\frac{\bar{t}}{b}\right)^3$	All edges simply supported	12
	Unflanged integrally stiffened	$\frac{N_x}{b\bar{\eta}E} = 0.970\left(\frac{\bar{t}}{b}\right)^{2.38}$	↓	↓
	Zee stiffened	$\frac{N_x}{b\bar{\eta}E} = 1.030\left(\frac{\bar{t}}{b}\right)^{2.36}$	↓	↓
	Truss-core sandwich	$\frac{N_x}{b\bar{\eta}E} = 1.108\left(\frac{\bar{t}}{b}\right)^2$	↓	↓
	Truss-core semisandwich	$\frac{N_x}{b\bar{\eta}E} = 0.59\left(\frac{\bar{t}}{b}\right)^2$	↓	↓
Multiweb box beams in bending	Unstiffened webs Unstiffened covers	$\Sigma = 1.87\left(\frac{M_i}{d^2 E}\right)^{3/7}$	Assumed webs carry no bending load and hinge joints between covers and webs	12
	Unstiffened webs Truss-core sandwich covers	$\Sigma = 2.11\left(\frac{M_i}{d^2 E}\right)^{1/2}$	↓	↓
	Truss-core sandwich webs Unstiffened covers	$\Sigma = 2.25\left(\frac{M_i}{d^2 E}\right)^{5/9}$	↓	↓
	Unf. int. stiffened webs Unstiffened covers	$\Sigma = 2.21\left(\frac{M_i}{d^2 E}\right)^{5/9}$	↓	↓
	Zee-stiffened webs Unstiffened covers	$\Sigma = 2.05\left(\frac{M_i}{d^2 E}\right)^{5/9}$	↓	↓
	Truss-core sandwich webs Truss-core sandwich covers	$\Sigma = 2.44\left(\frac{M_i}{d^2 E}\right)^{3/5}$	↓	↓
	Unf. int. stiffened webs Truss-core sandwich covers	$\Sigma = 2.40\left(\frac{M_i}{d^2 E}\right)^{3/5}$	↓	↓
	Zee-stiffened webs Truss-core sandwich covers	$\Sigma = 2.25\left(\frac{M_i}{d^2 E}\right)^{3/5}$	↓	↓

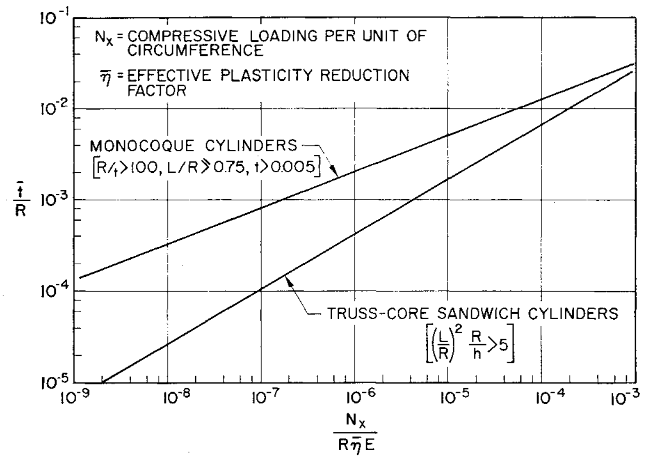


Fig. 4 Comparative efficiencies of monocoque and optimum truss-core sandwich, long cylinders subjected to uniform axial compression

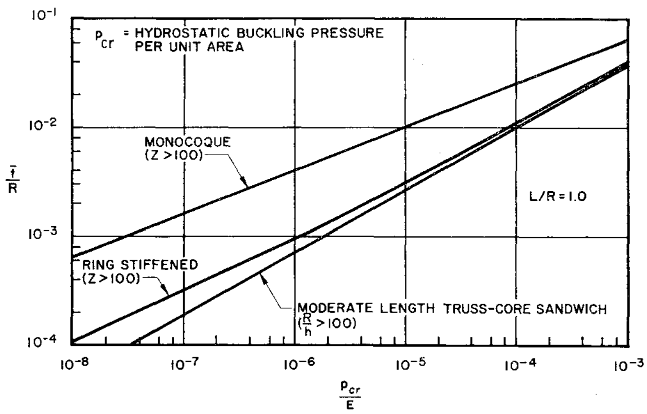


Fig. 5 Comparison of minimum weight-stiffened cylinder loaded elastically in hydrostatic compression

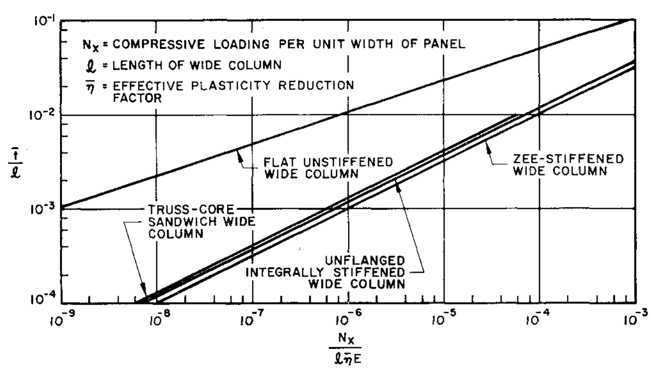


Fig. 6 Comparison of minimum weight envelopes of several types of stiffened, wide-column construction when subjected to a compression load in the direction of the stiffening elements

Because of its rapidly diminishing comparative superiority at stress levels above the proportional-limit stress and its comparatively high cost, the truss-core sandwich is recommended principally for low loading-index applications where stresses are elastic. This, in fact, may be a requirement in many designs where repeated loadings will be encountered and permanent deformation is undesirable.

Charts similar to Fig. 9 may be prepared readily for all configuration-loading-component combinations for which $\bar{\eta}$'s are presented here or are elsewhere available. For a complete set of figures applying to cross-rolled beryllium sheet (from which Fig. 9 has been taken), the reader is referred to Ref. 12. The minimum weight equations in Table 1 may be rewritten in terms of weight per unit of surface area of structural component, and these weights may be compared for equivalent structural shapes and loading indices to show relative efficiencies of various materials. The weight per unit of surface area is

$$W_i = \rho\bar{t} \tag{22}$$

where ρ is the density of the material.

For the long, axially compressed, truss-core sandwich cylinder, the efficiency equation becomes

$$N_x/R = (\mathcal{E}\bar{\eta}E/\rho^{5/3})(W_i/R)^{5/3} \tag{23}$$

Giving subscripts x and y to the unit weights and material properties in Eq. (23) to compare x and y materials, the ratio of weights to carry equal loadings elastically, with equal radii and efficiency factors, is

$$W_{i_x}/W_{i_y} = (\rho_x/\rho_y)(E_y/E_x)^{3/5} \tag{24}$$

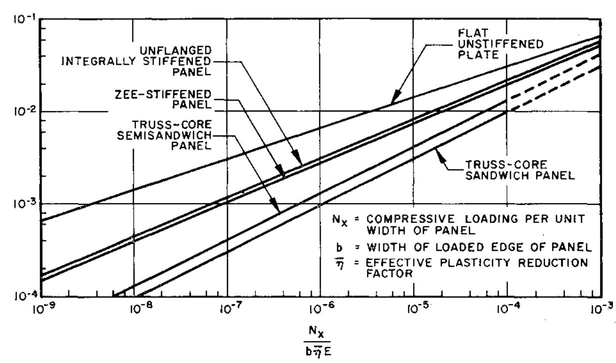
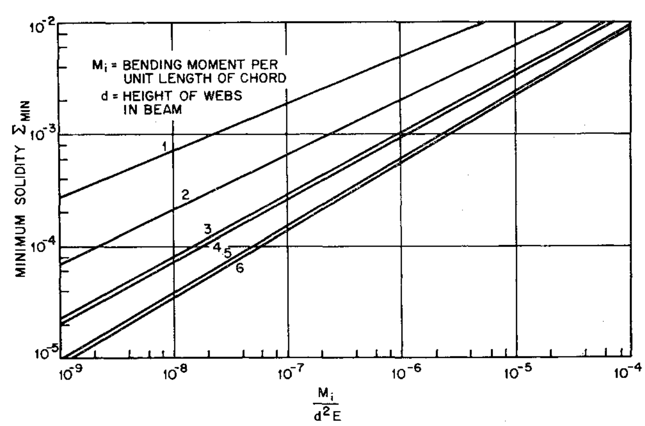


Fig. 7 Comparison of minimum weight envelopes of several types of stiffened panel construction when subjected to a compression load in the direction of the stiffened elements



CURVE	COVER PLATE	WEB
1	UNSTIFFENED PLATE	UNSTIFFENED PLATE
2	TRUSS-CORE SANDWICH	UNSTIFFENED PLATE
3	UNSTIFFENED PLATE	TRUSS-CORE SANDWICH OR UNFLANGED INTEGRALLY STIFFENED PLATE
4	UNSTIFFENED PLATE	ZEE-STIFFENED PLATE
5	TRUSS-CORE SANDWICH	TRUSS-CORE SANDWICH OR UNFLANGED INTEGRALLY STIFFENED PLATE
6	TRUSS-CORE SANDWICH	ZEE-STIFFENED PLATE

Fig. 8 Comparison of the elastic minimum-solidity equations for several multiweb box beams composed of different combinations of cover plate and web construction

Thus, the weight ratio for two different materials to perform equal functions elastically with equal efficiency factors is directly proportional to their density ratio and inversely proportional to their modulus ratio raised to the inverse of the exponent n in the efficiency equation. This conclusion is general and applies to all cases shown in Table 1. Figures 10 and 11 show weight ratios for some of the cases of Table 1 (those having $n = 3.0$ and 1.667, respectively) for various materials over a range of temperature, where, in each case, the y material is cross-rolled beryllium sheet [see Eq. (24)]. Figure 11 applies to the axially compressed truss-core sandwich cylinder of the foregoing example.

Figures 10 and 11 reveal that cross-rolled beryllium sheet is the lightest material for elastic applications in all configuration-loading-component combinations, except at temperatures exceeding approximately 1400°F. Further, as the exponent n decreases, the superiority of the beryllium sheet increases. The minimum advantage of cross-rolled beryllium sheet over its nearest competitor, magnesium, is seen to be 1:1.8 (Fig. 10). Thus, beryllium appears to have high potential in elastically stressed stability applications.

A materials comparison, including both plastic and elastic stresses, is shown in Fig. 12. This figure applies to flat, truss-core sandwich compression panels of optimum proportions at room temperature and is obtained by evaluating the minimum weight equation for this structure, when rewritten in the form of Eq. (23). Note that the graphical presentation technique used in Figs. 10 and 11 is not generally valid for determining weight ratios here, because $\bar{\eta}$ is not a constant above the proportional limit. The maximum stresses for all materials appearing in Fig. 12 have been taken to be equal to their respective compressive yield stresses.

On the basis of the data in Figs. 10 and 11, the superiority of cross-rolled beryllium sheet under elastic stress conditions is anticipated. However, Fig. 12 shows that this sheet remains superior, but to a lesser degree—even in the plastic stress range—to all materials investigated, except all-beta titanium (Ti-13V-11Cr-3Al). Charts in Ref. 12 of the type of Fig. 12, representing other configuration-loading-component combinations, show qualitatively the same results.

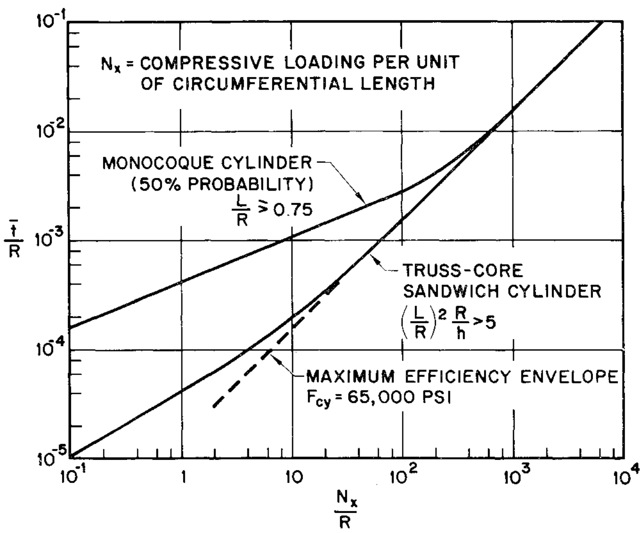


Fig. 9 Room-temperature minimum weight design chart for beryllium cross-rolled-sheet, long cylinders subjected to a uniform axial load

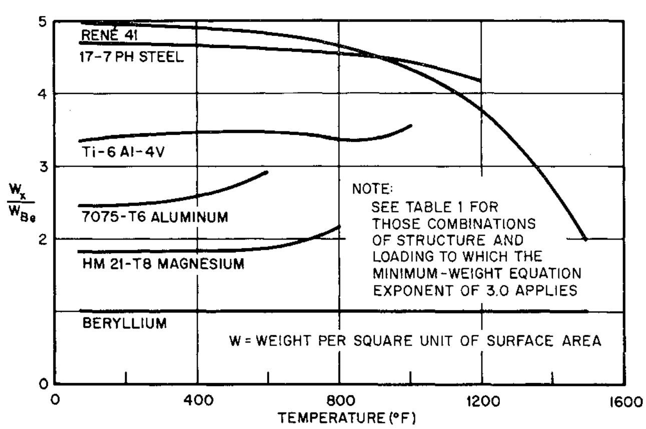


Fig. 10 Beryllium-based weight ratios for elastically loaded structures of optimum proportions having a minimum weight equation exponent of 3.0

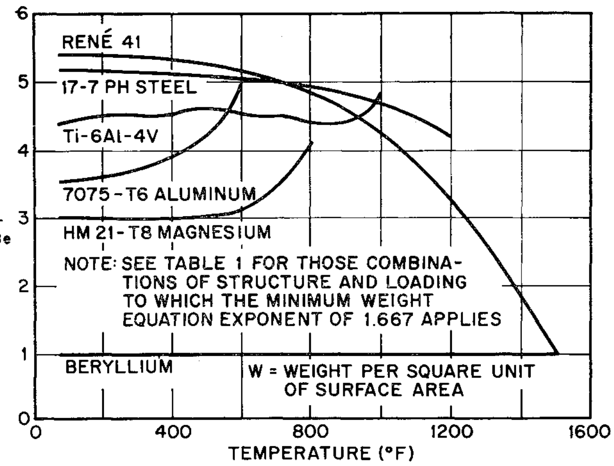


Fig. 11 Beryllium-based weight ratios for elastically loaded structures of optimum proportions having a minimum weight eqation exponent of 1.667

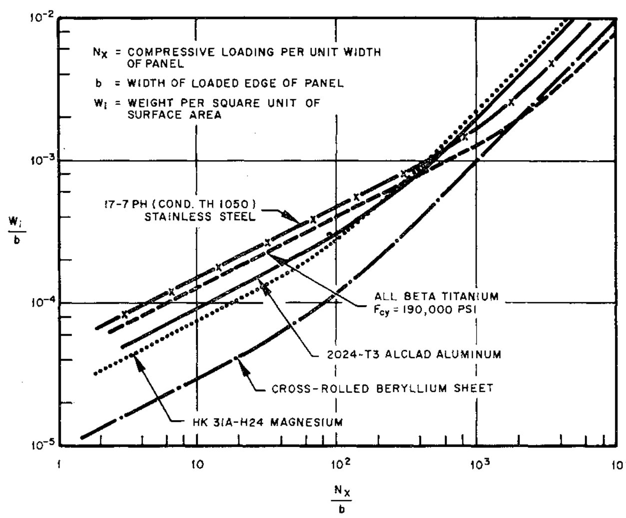


Fig. 12 Weight-load comparison at room temperature of structural materials used in truss-core sandwich compression panels of optimum proportions

Conclusions and Recommendations

The existing principles and methods of minimum weight analysis can be applied readily to a variety of configuration-loading-component combinations. The resulting minimum weight design information can be put in a convenient form for design use. The present information is characteristic of what may be anticipated in future analyses of this type.

Minimum weight design information for many new configuration-loading-component combinations has been presented. Because of the idealization assumed in its derivation, the design information will be of principal interest in preliminary design, where the basic structural configurations will be established.

Sandwich structures, of which the truss-core sandwich has been selected as representative, are shown to be very efficient load-carrying members for all loading-component combinations treated in this article, except as wide columns—particularly at low to moderate values of the various loading indexes.

Cross-rolled beryllium sheet is the most efficient material for all configuration-loading-component combinations investigated, when elastically stressed in the lower temperature range. In applications involving plastic stresses, beryllium is competitive with the so-called high-strength materials.

The validity of small-deflection theory for predicting the behavior of axially compressed, long, truss-core sandwich circular cylindrical shells has been verified for a portion of the practical design range by some independently conducted tests. Further experimental verification of the theoretical minimum weight analyses presented here is required, particularly for structures such as the axially compressed cylinder, where controversy exists over the proper theory for predicting their buckling behavior. Minimum weight studies of other stiffened structural components, such as conical shells and spherical caps, are needed. In addition, the minimum weight of structures under combined loading is of interest and should be determined.

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APRIL 1963 AIAA JOURNAL VOL. 1, NO. 4

Conical Segment Method for Analyzing Open Crown Shells of Revolution for Edge Loading

Robert R. Meyer* and Marilyn B. Harmon†
Douglas Aircraft Company Inc., Santa Monica, Calif.

A solution, accurate, rapid, simple enough for design use, and valid for all regions, has been obtained for the stress distribution and influence coefficients for a variable thickness shell of revolution formed by a generator of arbitrary shape. The shell is subdivided into a series of equivalent conical segments whose individual thicknesses are the local segment average. Conditions of continuity then are applied at the boundaries of each conical segment to evaluate the indeterminate edge shears and moments using digital equipment. Influence coefficient comparisons for a wide range of shell geometries are made between the cone solution and solutions by other methods from the literature and show agreement within 4%. The cone solution reciprocity relations are shown to be valid to five significant figures. Limiting conditions indicate that good approximations of the influence coefficients and the stresses can be obtained by using 10 cones in most cases.

Nomenclature

E = modulus of elasticity
ν = Poisson's ratio
t = thickness
Δz = altitude of a truncated conical segment
R = radius of curvature of the median surface of the cone measured in the truncating plane
α = angle between the axis of revolution and the generatrix of the conical segment
X_i = load or moment applied to an edge of a conical segment
δ_{ik} = deflection at i in the direction of load X_i due to a unit load at k (displacement or rotation), influence coefficient
a = distance from the shell axis of rotation to the center of the radius of curvature for a toroidal shell
b = maximum shell cross-section dimension, i.e., toroidal radius or major axis of an ellipse
Q_s = shear force on the meridian plane
N_s = normal force on the meridian plane
M_s = bending moment on the meridian plane

Received by IAS June 5, 1962; revision received January 18, 1963. The authors wish to thank Barbara Freeman and Ralph Kahn of the Space Computing Engineering Section, Douglas Aircraft Company Inc. for programming the equations in this article for the IBM 7090 computer.

* Space Strength Section, Missile and Space Systems Division.
† Space Strength Section, Missile and Space Systems Division.